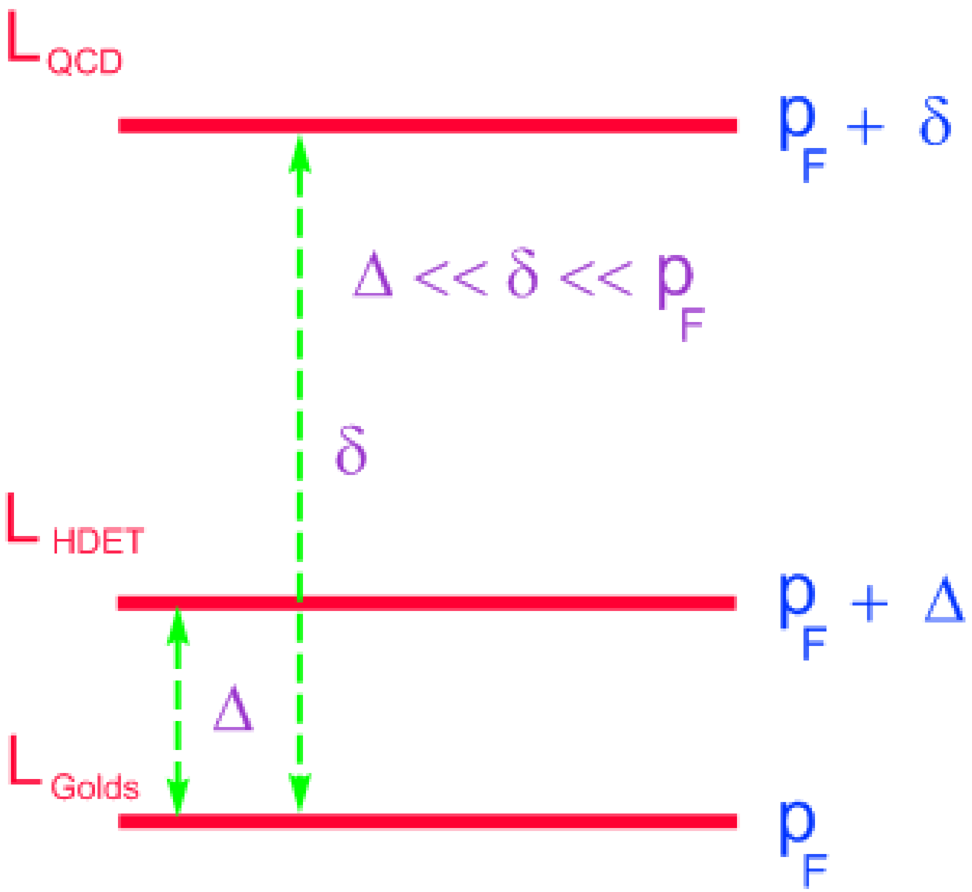
NGB and their parameters

- Gradient expansion: parameters of the NGB's
- Masses of the NGB's
- The role of the chemical potential for scalar fields: BE condensation
- Dispersion relations for the gluons

Hierarchies of effective lagrangians



Integrating out heavy degrees of freedom we have two scales. The gap Δ and a cutoff, δ above which we integrate out. Therefore: two different effective theories, L_{HDET} and L_{Golds}

Gradient expansion: NGB's parameters

Recall from HDET that in the CFL phase

$$\boxed{L_D = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \begin{bmatrix} iV \cdot D_{AB} & -\Delta_{AB} \\ -\Delta^*_{AB} & i\tilde{V} \cdot D_{AB} \end{bmatrix} \chi^B + (L \leftrightarrow R)}$$

and in the basis

$$\chi_i^\alpha = \frac{1}{\sqrt{2}} \sum_{A=1}^{9} (\lambda_A)_i^\alpha \chi^A$$

$$\boxed{\Delta_{AB} = \Delta_A \delta_{AB}}$$

$$\Delta_A = \begin{cases} A=1,\cdots,8 & \Delta_A = \Delta \\ A = 9 & \Delta_9 = -2\Delta \end{cases}$$

$$\boxed{L_D = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \begin{bmatrix} iV\cdot\partial & -\Delta_A \\ -\Delta_A & i\tilde{V}\cdot\partial \end{bmatrix} \chi^A + (L \leftrightarrow R)}$$

Propagator

$$S_{AB} = \frac{\delta_{AB}}{V\cdot\ell\,\tilde{V}\cdot\ell - \Delta_A^2} \begin{bmatrix} \tilde{V}\cdot\ell & \Delta_A \\ \Delta_A & V\cdot\ell \end{bmatrix}$$

Coupling to the U(1) NGB:

$$U = e^{i\sigma/f_\sigma}, \quad V = e^{i\tau/f_\tau}$$

$$\sigma = f_\sigma \varphi, \qquad \tau = f_\tau \theta$$

$$\psi_L \to e^{i(\alpha+\beta)}\psi_L, \quad \psi_R \to e^{i(\alpha-\beta)}\psi_R$$

$$U \to e^{-i\alpha}U, \qquad V \to e^{-i\beta}V$$

Consider now the case of the $U(1)_B$ NGB. The invariant Lagrangian is:

$$\boxed{L_D = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \begin{bmatrix} iV\cdot\partial & -U^{\dagger 2}\Delta_A \\ -U^2\Delta_A & i\tilde{V}\cdot\partial \end{bmatrix} \chi^A}$$

At the lowest order in σ

$$L_\sigma = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \Delta_A \begin{bmatrix} 0 & \dfrac{2i\sigma}{f_\sigma} + \dfrac{2\sigma^2}{f_\sigma^2} \\ -\dfrac{2i\sigma}{f_\sigma} + \dfrac{2\sigma^2}{f_\sigma^2} & 0 \end{bmatrix} \chi^A$$

generates 3-linear and 4-linear couplings

Generating functional: $Z[\sigma] = \int D\chi D\chi^\dagger e^{i\int \chi^\dagger A(\sigma)\chi}$

$$A(\sigma) = S_0^{-1} + \frac{2i\sigma\Delta_A}{f_\sigma}\Gamma_0 + \frac{2\sigma^2\Delta_A}{f_\sigma^2}\Gamma_1$$

$$S_0^{-1} = \begin{bmatrix} V\cdot\ell & -\Delta_A \\ -\Delta_A & \tilde{V}\cdot\ell \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

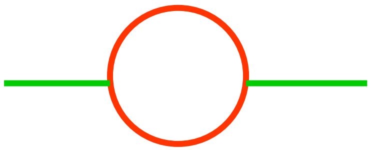
$$Z[\sigma] = (\det[A(\sigma)])^{1/2} = e^{\frac{1}{2}\mathrm{Tr}[\log A(\sigma)]}$$

$$\boxed{S_{\mathrm{eff}}[\sigma] = -\frac{i}{2}\mathrm{Tr}[\log A(\sigma)]}$$

$$-i\mathrm{Tr}[\log A(\sigma)] = -i\mathrm{Tr}\left[\log\left(S^{-1}\left(1+S\frac{2i\sigma\Delta}{f_\sigma}\Gamma_0 + S\frac{2\sigma^2\Delta}{f_\sigma^2}\Gamma_1\right)\right)\right] =$$

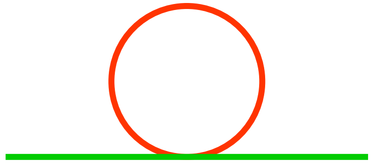
$$= -i\mathrm{Tr}\left[\log S^{-1}\right] - i\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}\left(iS\frac{2i\sigma\Delta}{f_\sigma}i\Gamma_0 + iS\frac{2\sigma^2\Delta}{f_\sigma^2}i\Gamma_1\right)^n$$

At the lowest order:



$$S_{\mathrm{eff}} = \frac{i}{4}\int dx dy \mathrm{Tr}\left[\frac{iS(y,x)2i\sigma(x)\Delta}{f_\sigma}i\Gamma_0\frac{iS(x,y)2i\sigma(y)\Delta}{f_\sigma}i\Gamma_0\right] +$$

$$+\frac{i}{2}\int dx \mathrm{Tr}\left[\frac{iS(x,x)2\sigma^2(x)\Delta}{f_\sigma^2}i\Gamma_1\right]$$



Feynman rules

- For each fermionic internal line

$$iS_{AB} = i\delta_{AB} S(p) = \frac{i\delta_{AB}}{V\cdot\ell\,\tilde{V}\cdot\ell - \Delta_A^2} \begin{bmatrix} \tilde{V}\cdot\ell & \Delta_A \\ \Delta_A & V\cdot\ell \end{bmatrix}$$

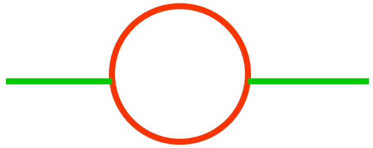
- For each vertex a term iL_{int}

- For each internal momentum not constrained by momentum conservation:

$$\frac{4\pi\mu^2}{(2\pi)^4} \int d^2\ell = \frac{\mu^2}{4\pi^3} \int_{-\delta}^{+\delta} d\ell_{\parallel} \int_{-\infty}^{+\infty} d\ell_0$$

- Factor $2x(-1)$ from Fermi statistics and spin. A factor $1/2$ from replica trick.

- A statistical factor when needed.



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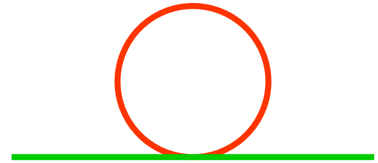


$$iL_{eff} = iL_I(p) + iL_{II}(p) = -\frac{1}{2}\int \frac{d\vec{v}}{4\pi}\sum_A \frac{\mu^2\Delta_A^2}{\pi^3 f_\sigma^2}\times$$

$$\times\int d^2\ell\left[\frac{\tilde{V}\cdot(\ell+p)\sigma V\cdot\ell\sigma + V\cdot(\ell+p)\sigma\tilde{V}\cdot\ell\sigma - 2\Delta_A^2\sigma^2}{D_A(\ell+p)D_A(\ell)} - \frac{2\sigma^2}{D_A(\ell)}\right]$$

$$D_A(\ell) = V\cdot\ell\tilde{V}\cdot\ell - \Delta_A^2 + i\varepsilon$$

Goldstone theorem: $$\boxed{L_I(0)+L_{II}(0)=0}$$

Expanding in p/Δ:

$$L_{\mathrm{eff}}(x) = \frac{9\mu^2}{\pi^2 f_\sigma^2}\frac{1}{2}\int \frac{d\vec{v}}{4\pi}(V\cdot\partial)\sigma(x)(\tilde{V}\cdot\partial)\sigma(x)$$

$$\int \frac{d\vec{v}}{4\pi} V^\mu \tilde{V}^\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} \end{bmatrix}$$

$$\boxed{\begin{gathered} L_{\mathrm{eff}}(x) = \frac{1}{2}\frac{9\mu^2}{\pi^2 f_\sigma^2}\left[(\partial_0\sigma)^2 - v_\sigma^2\left(\vec{\nabla}\sigma\right)^2\right] \\ v_\sigma^2 = \frac{1}{3}, \quad f_\sigma^2 = \frac{9\mu^2}{\pi^2} \end{gathered}}$$

CFL

For the V NGB same result in CFL, whereas in 2SC

$$\boxed{v_\tau^2 = \frac{1}{3}, \quad f_\tau^2 = \frac{4\mu^2}{\pi^2}}$$

2SC

With an analogous calculation:

$$L_{eff} = \frac{\mu^2(21-8\log 2)}{36\pi^2 F^2}\frac{1}{2}\sum_{a=1}^{8}\left[(\partial_0\Pi^a)^2 - v^2\,|\vec{\nabla}\Pi^a|^2\right]$$

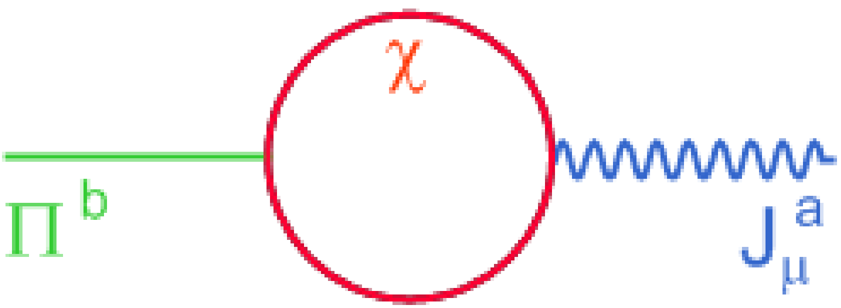
$$\boxed{v^2 = \frac{1}{3}, \quad F^2 = F_T^2 = \frac{\mu^2(21-8\log 2)}{36\pi^2}}$$

Dispersion relation for the NGB's

$$\boxed{E = \pm \frac{1}{\sqrt{3}} |\vec{p}|}$$

Different way of computing:

$$\langle 0 | J_\mu^a | \Pi^b \rangle = iF\delta^{ab}\tilde{p}_\mu, \quad \tilde{p}_\mu = \left(p^0, \frac{1}{3}\vec{p} \right)$$



Current conservation:

$$\boxed{p \cdot \tilde{p} = E^2 - \frac{1}{3} |\vec{p}|^2 = 0}$$

Masses of the NGB's

QCD mass term: $\bar{\psi}_L M \psi_R + \mathrm{h.c.}$

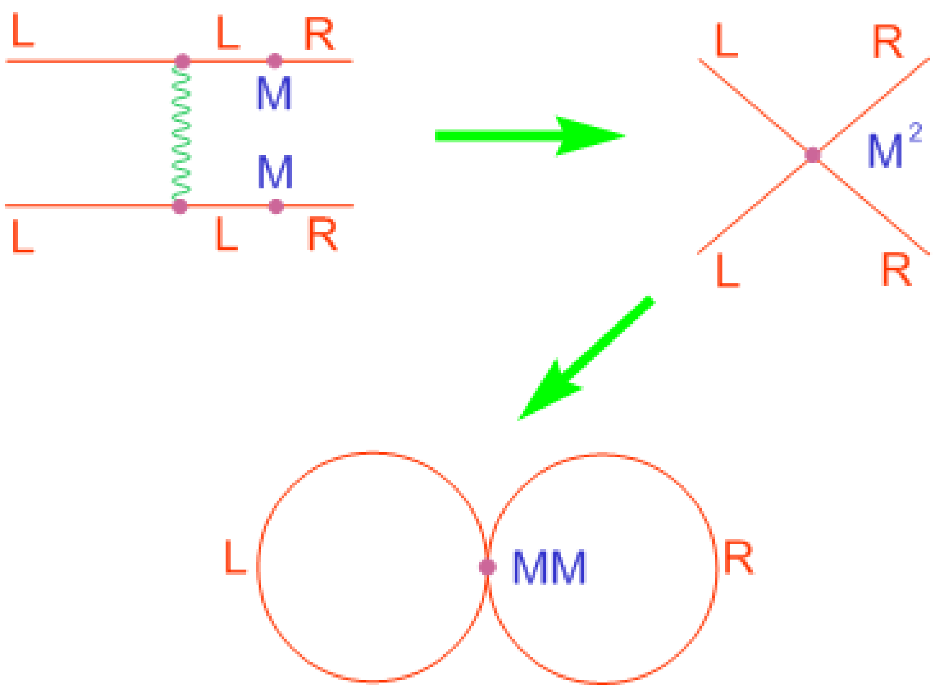
$$M \xrightarrow[Z_{2L}]{} -M$$

$$\tilde{\Sigma} = (Y^\dagger X)^T = e^{4i\theta}\Sigma^T$$

$$L_{\mathrm{masses}} = -c\left(\det[M]\mathrm{Tr}[M^{-1}\tilde{\Sigma} + \mathrm{h.c.}\right) - c'\left(\det(\tilde{\Sigma})\mathrm{Tr}[(M\tilde{\Sigma}^\dagger)^2]\right) -$$
$$-c''\left(\mathrm{Tr}[M\tilde{\Sigma}^\dagger]\mathrm{Tr}[M^\dagger\tilde{\Sigma}]\right)$$

Calculation of the coefficients from QCD

Mass insertion in QCD



Effective 4-fermi

$$c = \frac{3\Delta^2}{2\pi^2}, c' = 0, c'' = 0$$

Contribution to the vacuum energy

Consider:

$$L_{QCD} = \bar{\psi}(i\not{D} + \mu\gamma_0)\psi - \bar{\psi}_L M \psi_R - \bar{\psi}_R M \psi_L - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a}$$

Solving for $\psi_{-,L}$ as in HDET

$$\psi_{-,L} = \frac{1}{2\mu}\left(-i\gamma_0 \not{D}_\perp \psi_{+,L} + \gamma_0 M \psi_{+,R}\right)$$

like chemical potential

$$L_D = \psi^\dagger_{+,L} i V_\mu \left(D^\mu + \frac{i}{2\mu} M M^\dagger g^{\mu 0} \right) \psi_{+,L} - \frac{1}{2\mu} \psi^\dagger_{+,L} (\not{D}_\perp)^2 \psi_{+,L} +$$

$$+\left(L \to R, M \to M^\dagger\right) + \cdots$$

Consider fermions at finite density:

$$L = \bar{\psi} i \gamma_\nu \left(\partial^\nu - i \mu g^{\nu 0} \right) \psi$$

as a gauge field A^0

Invariant under: $\psi \to e^{i\alpha(t)} \psi, \quad \mu \to \mu + \dot{\alpha}(t)$

Define: $X_L = \frac{1}{2\mu} M M^\dagger, \quad X_R = \frac{1}{2\mu} M^\dagger M$

Invariance under:

$$\psi_{+,L} \to L(t) \psi_{+,L}, \quad \psi_{+,R} \to R(t) \psi_{+,R},$$

$$X_L \to L(t) X_L L^\dagger(t) + i L(t) \partial_0 L^\dagger(t),$$

$$X_R \to R(t) X_R R^\dagger(t) + i R(t) \partial_0 R^\dagger(t)$$

The same symmetry should hold at the level of the effective theory for the CFL phase (NGB's), implying that

$$\boxed{\partial_0\Sigma \to \nabla_0\Sigma = \partial_0\Sigma + i\Sigma\left(\frac{MM^\dagger}{2\mu}\right)^T - i\left(\frac{M^\dagger M}{2\mu}\right)^T\Sigma}$$

The generic term in the derivative expansion of the NGB effective lagrangian has the form

$$\boxed{L_{NGB} \approx F^2\Delta^2\left(\frac{\partial_0 + iMM^\dagger/2\mu}{\Delta}\right)^n\left(\frac{\vec{\nabla}}{\Delta}\right)^m\left(\frac{M^2}{F^2}\right)^p\Sigma^q\Sigma^{\dagger r}}$$

$$L_{NGB} \approx F^2\Delta^2 \left(\frac{\partial_0 + iMM^\dagger/2\mu}{\Delta}\right)^n \left(\frac{\vec{\nabla}}{\Delta}\right)^m \left(\frac{M^2}{F^2}\right)^p \Sigma^q \Sigma^{\dagger r}$$

Compare the two contribution to quark masses:

kinetic term

$$F^2\Delta^2 \frac{m^4}{\mu^2\Delta^2}\frac{1}{F^2} \approx \frac{m^4}{\mu^2}$$

mass insertion

$$F^2\Delta^2 \frac{m^2}{F^2}\frac{1}{F^2} \approx \frac{\Delta^2 m^2}{F^2}$$

Same order of magnitude for $m \approx \Delta$ **since** $F \approx \mu$

The role of the chemical potential for scalar fields: Bose-Einstein condensation

- A conserved current may be coupled to the a gauge field.
- Chemical potential is coupled to a conserved charge.
- The chemical potential must enter as the fourth component of a gauge field.

Complex scalar field:

$$L = \left(\partial_0 + i\mu\right)\varphi^\dagger\left(\partial_0 - i\mu\right)\varphi - \left(\vec{\nabla}\varphi^\dagger\right)\cdot\left(\vec{\nabla}\varphi\right) - m^2\varphi^\dagger\varphi - \lambda\left(\varphi^\dagger\varphi\right)^2 =$$

$$= \partial_\mu\varphi^\dagger\partial^\mu\varphi - \left(m^2 - \mu^2\right)\varphi^\dagger\varphi - \lambda\left(\varphi^\dagger\varphi\right)^2 + i\mu\left(\varphi^\dagger\partial_0\varphi - \partial_0\varphi^\dagger\varphi\right)$$

negative mass term

breaks C

Mass spectrum:

$$p^2 - (m^2 - \mu^2) + 2\mu Q p_0 = 0 \quad (Q = \pm 1)$$

$$\Downarrow$$

$$(E + \mu Q)^2 = m^2 + |\vec{p}\,|^2$$

For $\mu < m$

$$\boxed{m_{P,\bar{P}} = \mp\mu + m}$$

At $\mu = m$, second order phase transition.
Formation of a condensate obtained from:

$$V = \left(m^2 - \mu^2\right)\varphi^\dagger\varphi + \lambda\left(\varphi^\dagger\varphi\right)^2$$

$$\Downarrow$$

$$\left\langle \varphi^\dagger\varphi \right\rangle = \frac{\mu^2 - m^2}{2\lambda} \quad (\mu > m)$$

Charge density

$$\rho = -\frac{\partial V}{\partial \mu} = 2\mu\left\langle \varphi^\dagger\varphi \right\rangle = \frac{\mu}{\lambda}\left(\mu^2 - m^2\right)$$

Ground state = Bose-Einstein condensate

$$\langle\varphi\rangle = \frac{v}{\sqrt{2}}, \quad v^2 = \frac{\mu^2 - m^2}{\lambda} \qquad \varphi(x) = \frac{1}{\sqrt{2}}\left(v + h(x)\right)e^{i\theta(x)/v}$$

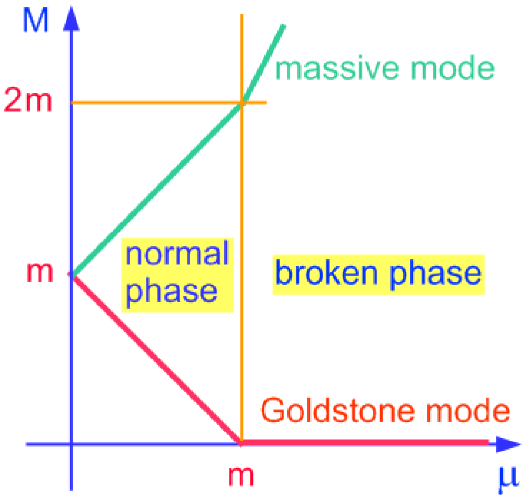
$$L_2 = \frac{1}{2}\partial_\mu\theta\partial^\mu\theta + \frac{1}{2}\partial_\mu h\partial^\mu h - \lambda v^2 h^2 - 2\mu h\partial_0\theta$$

Mass spectrum

$$\det\begin{bmatrix} p^2 - 2\lambda v^2 & 2i\mu E \\ -2i\mu E & p^2 \end{bmatrix} = 0$$

At zero momentum

$$M^2\left(M^2 - 2\lambda v^2 - 4\mu^2\right) = 0$$



$$\boxed{\begin{gathered} M^2 = 0 \\ M^2 = 6\mu^2 - 2m^2 \end{gathered}}$$

At small momentum

$$E_{NGB} \approx \sqrt{\frac{\mu^2 - m^2}{3\mu^2 - m^2}}\,|\vec{p}\,|$$

$$E_{massive} \approx \sqrt{6\mu^2 - 2m^2 + \frac{9\mu^2 - m^2}{6\mu^2 - m^2}\,|\vec{p}\,|^2}$$

$$\boxed{v_{NGB}^2 = \frac{\mu^2 - m^2}{3\mu^2 - m^2} \xrightarrow[\mu \to \infty]{} \frac{1}{3}}$$

Back to CFL. From the structure

$$\boxed{m_{P,\bar{P}} = \mp\mu + m}$$

$$\boxed{\begin{aligned} m_{\pi^\pm} &= \mp\frac{m_d^2 - m_u^2}{2\mu} + \sqrt{\frac{2c}{F^2}(m_u + m_d)m_s}\,, \\ m_{K^\pm} &= \mp\frac{m_s^2 - m_u^2}{2\mu} + \sqrt{\frac{2c}{F^2}(m_u + m_s)m_d}\,, \\ m_{K^0,\bar{K}^0} &= \mp\frac{m_s^2 - m_d^2}{2\mu} + \sqrt{\frac{2c}{F^2}(m_d + m_s)m_u} \end{aligned}}$$

$$c = \frac{3\Delta^2}{2\pi^2}$$

$$F^2 = \frac{\mu^2(21 - 8\log 2)}{36\pi^2}$$

First term from "chemical potential" like $MM^\dagger$ kinetic term, the second from mass insertions

For large values of m_{s}:

$$m_{\pi^{\pm}} = \sqrt{\frac{2c}{F^2}(m_u + m_d)m_s},$$

$$m_{K^{\pm}} = \mp\frac{m_s^2}{2\mu} + \sqrt{\frac{2c}{F^2}m_s m_d}, \quad m_{K^0,\bar{K}^0} = \mp\frac{m_s^2}{2\mu} + \sqrt{\frac{2c}{F^2}m_s m_u}$$

and the masses of K^{+} and K^{0} are pushed down. For the critical value

$$m_s\big|_{crit} = \left(\frac{12\mu^2}{\pi^2 F^2}\right)^{1/3} \sqrt[3]{m_{u,d}\Delta^2} \approx 3.03\sqrt[3]{m_{u,d}\Delta^2},$$

masses vanish

$$m_s\big|_{crit} \approx 40 - 110\ \text{MeV}$$

For larger values of m_s these modes become unstable. Signal of condensation. Look for a kaon condensate of the type:

$$\Sigma = e^{i\alpha\lambda_4} = 1 + (\cos\alpha - 1)\lambda_4^2 + i\lambda_4 \sin\alpha$$

(In the CFL vacuum, $\Sigma = 1$) and substitute inside the effective lagrangian

$$V(\alpha) = F^2\left(-\frac{1}{2}\left(\frac{m_s^2}{2\mu}\right)\sin^2\alpha + \frac{2cm_s m}{F^2}(1-\cos\alpha)\right)$$

negative contribution from the "chemical potential"

positive contribution from mass insertion

Defining

$$\mu_{\mathrm{eff}} = \frac{m_s^2}{2\mu}, \quad \left(m_K^0\right)^2 = \frac{2 c m_s m}{F^2}$$

$$V(\alpha) = F^2\left(-\frac{1}{2}\mu_{\mathrm{eff}}^2 \sin^2\alpha + (m_K^0)^2(1-\cos\alpha)\right)$$

with solution

$$\boxed{\cos\alpha = \frac{\left(m_K^0\right)^2}{\mu_{\mathrm{eff}}^2}, \quad \mu_{\mathrm{eff}} > m_K^0}$$

and hypercharge density

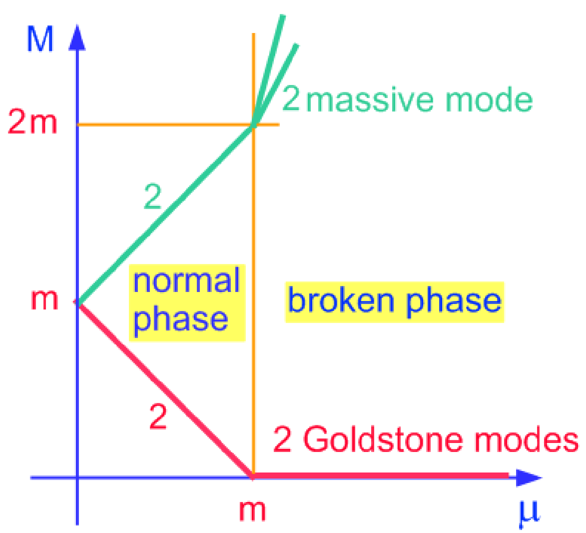
$$n_Y = -\frac{\partial V}{\partial \mu_{\mathrm{eff}}} = \mu_{\mathrm{eff}} F^2\left(1 - \frac{\left(m_K^0\right)^4}{\mu_{\mathrm{eff}}^4}\right)$$

Mass terms break original $SU(3)_{c+L+R}$ to $SU(2)_I x U(1)_Y$. Kaon condensation breaks this to $U(1)$

$$Q = \frac{1}{2}\left(\lambda_3 - \frac{1}{\sqrt{3}}\lambda_8\right), \quad [Q, \Sigma] = 0$$

$$\Sigma \xrightarrow[SU(2)_I \otimes U(1)_Y]{} (\pi^\pm, \pi^0) \oplus \underbrace{(K^+, K^0)} \oplus (\bar{K}^0, K^-) \oplus (\eta)$$

breaking through the doublet as in the SM



Only 2 NGB's from K^0, K^+ instead of expected 3 (see Chada & Nielsen 1976)

Chada and Nielsen theorem: The number of NGB's depends on their dispersion relation

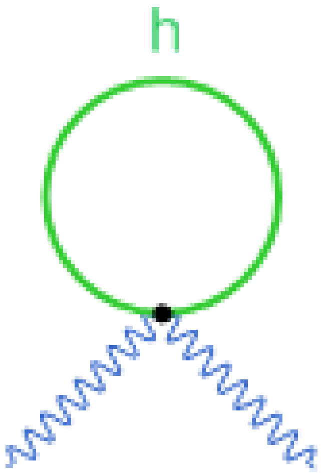
I. If E is linear in k, one NGB for any broken symmetry

II. If E is quadratic in k, one NGB for any two broken generators

In relativistic case always of type I, in the non-relativistic case both possibilities arise, for instance in the ferromagnet there is one NGB of type II, whereas for the antiferromagnet there are two NGB's of type I

Dispersion relations for the gluons

The bare Meissner mass



The heavy field contribution comes from the term

$$-\psi^{\dagger}_{h+}\frac{\left(\not{D}_{\perp}\right)^{2}}{2\mu+i\tilde{V}\cdot D}\psi_{h+}=-P^{\mu\nu}\psi^{\dagger}_{h+}\frac{D_{\mu}D_{\nu}}{2\mu+i\tilde{V}\cdot D}\psi_{h+}$$

$$P^{\mu\nu}=g^{\mu\nu}-\frac{1}{2}\left[V^{\mu}\tilde{V}^{\nu}+V^{\nu}\tilde{V}^{\mu}\right]$$

Notice that the first quantized hamiltonian is:

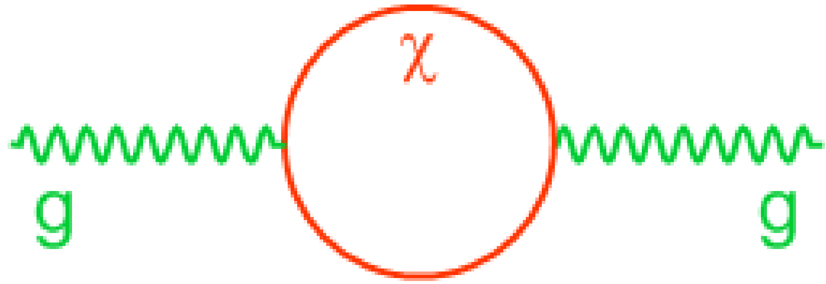
$$H = \left|\vec{p} - g\vec{A}\right| + eA_0 \approx |\vec{p}| + gA_0 - g\vec{v}\cdot\vec{A} + \underbrace{\frac{g^2}{2|\vec{p}|}\left(|\vec{A}|^2 - (\vec{v}\cdot\vec{A})^2\right)}$$

Since the zero momentum propagator is the density one gets

$$g^2 \times \underset{\text{spin}}{2} \times N_f \times \int_{|\vec{p}|<\mu} \frac{d^3\vec{p}}{(2\pi)^3}\frac{1}{2|\vec{p}|}\mathrm{Tr}\left[\vec{A}^2 - \frac{(\vec{p}\cdot\vec{A})^2}{|\vec{p}|^2}\right]$$

$$N_f\frac{g^2\mu^2}{6\pi^2}\frac{1}{2}\sum_a \vec{A}^a\cdot\vec{A}^a = \frac{1}{2}m_{BM}^2\sum_a \vec{A}^a\cdot\vec{A}^a, \quad m_{BM}^2 = N_f\frac{g^2\mu^2}{6\pi^2}$$

Gluons self-energy



Vertices from $igA_\mu^a J_a^\mu$

Consider first 2SC for the unbroken gluons:

$$\Pi_{ab}^{00}(p) = \delta_{ab}\frac{g^2\mu^2}{18\pi^2\Delta^2}|\vec{p}|^2,$$

$$\Pi_{ab}^{kl}(p) = \Pi_{ab}^{kl,self}(p) + \Pi_{ab}^{kl}(p) =$$

from m_{BM}^2

$$= \delta_{ab}\delta^{kl}\frac{g^2\mu^2}{3\pi^2}\left(1+\frac{p_0^2}{6\Delta^2}\right) - \delta_{ab}\delta^{kl}\frac{g^2\mu^2}{3\pi^2} = \delta_{ab}\delta^{kl}\frac{g^2\mu^2}{18\pi^2\Delta^2}p_0^2,$$

$$\Pi_{ab}^{0k}(p) = \delta_{ab}\frac{g^2\mu^2}{18\pi^2\Delta^2}p^0 p^k$$

- Bare Meissner mass cancels out the constant contribution from the s.e.

- All the components of the vacuum polarization have the same wave function renormalization

$$L = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{1}{2}\Pi_{ab}^{\mu\nu}A_\mu^a A_\nu^b = \frac{1}{2}\left(E_i^a E_i^a - B_i^a B_i^a\right) + \frac{k}{2}E_i^a E_i^a$$

$$k = \frac{g^2\mu^2}{18\pi^2\Delta^2}$$

Dielectric constant $\varepsilon = k+1$, and magnetic permeability $\lambda = 1$

$$v = \frac{1}{\sqrt{\varepsilon\lambda}} \approx \frac{\Delta}{g\mu}$$

Broken gluons

a	$\Pi^{00}(0)$	$-\Pi^{ij}(0)$
1-3	0	0
4-7	$3m_g^2/2$	$m_g^2/2$
8	$3m_g^2$	$m_g^2/3$

$$\boxed{m_g^2 = \frac{g^2\mu^2}{3\pi^2}}$$

But physical masses depend on the wave function renormalization

$$\approx \frac{g^2\mu^2}{\Delta^2}$$

Rest mass defined as the energy at zero momentum:

$$m_R = \sqrt{2}\Delta, \quad a = 4,5,6,7$$

$$m_R = \frac{g\mu}{\pi}, \qquad a = 8$$

The expansion in p/Δ cannot be trusted, but numerically

$$m_R \approx 0.9\Delta, \quad a = 4,5,6,7$$

In the CFL case one finds:

$$m_D^2 = \frac{g^2\mu^2}{36\pi^2}(21 - 8\log 2) = g^2F^2$$

$$m_M^2 = \frac{g^2\mu^2}{\pi^2}\left(-\frac{11}{36} - \frac{2}{27}\log 2 + \underbrace{\frac{1}{2}}_{\text{from bare Meissner mass}}\right) = \frac{m_D^2}{3}$$

Recall that from the effective lagrangian we got:

$$m_D^2 = \alpha_T g^2 F_T^2, \quad m_M^2 = \alpha_S v^2 g^2 F_T^2$$

implying $\alpha_S = \alpha_T = 1$ and fixing all the parameters.

We find: $$m_R \approx \frac{m_D}{\sqrt{3\alpha_1}}, \quad \alpha_1 = \frac{g^2\mu^2}{216\pi^2\Delta^2}\left(7+\frac{16}{3}\log 2\right)$$

$$\Downarrow$$

$$m_R \approx 1.70\Delta$$

Numerically $$\boxed{m_R \approx 1.36\Delta}$$

Different quark masses

We have seen that for one massless flavors and a massive one (m_s), the condensate may be disrupted for

$$\mu < \frac{m_s^2}{2\Delta}$$

The radii of the Fermi spheres are:

$$p_{F_1} = \sqrt{\mu^2 - m_s^2} \approx \mu - \frac{m_s^2}{2\mu}, \quad p_{F_2} = \mu$$

As if the two quarks had different chemical potential ($m_s^2/2\mu$)

Simulate the problem with two massless quarks with different chemical potentials:

$$\mu_u = \mu + \delta\mu, \quad \mu_d = \mu - \delta\mu$$

$$\mu = \frac{\mu_u + \mu_d}{2}, \quad \delta\mu = \frac{\mu_u - \mu_d}{2}$$

Can be described by an interaction hamiltonian

$$\boxed{H_I = -\delta\mu\,\psi^\dagger\sigma_3\psi}$$

Lot of attention in normal SC.

H_I changes the inverse propagator

$$S_0^{-1} = \begin{bmatrix} V\cdot\ell + \delta\mu\,\sigma_3 & -\Delta \\ -\Delta^* & \tilde{V}\cdot\ell + \delta\mu\,\sigma_3 \end{bmatrix}$$

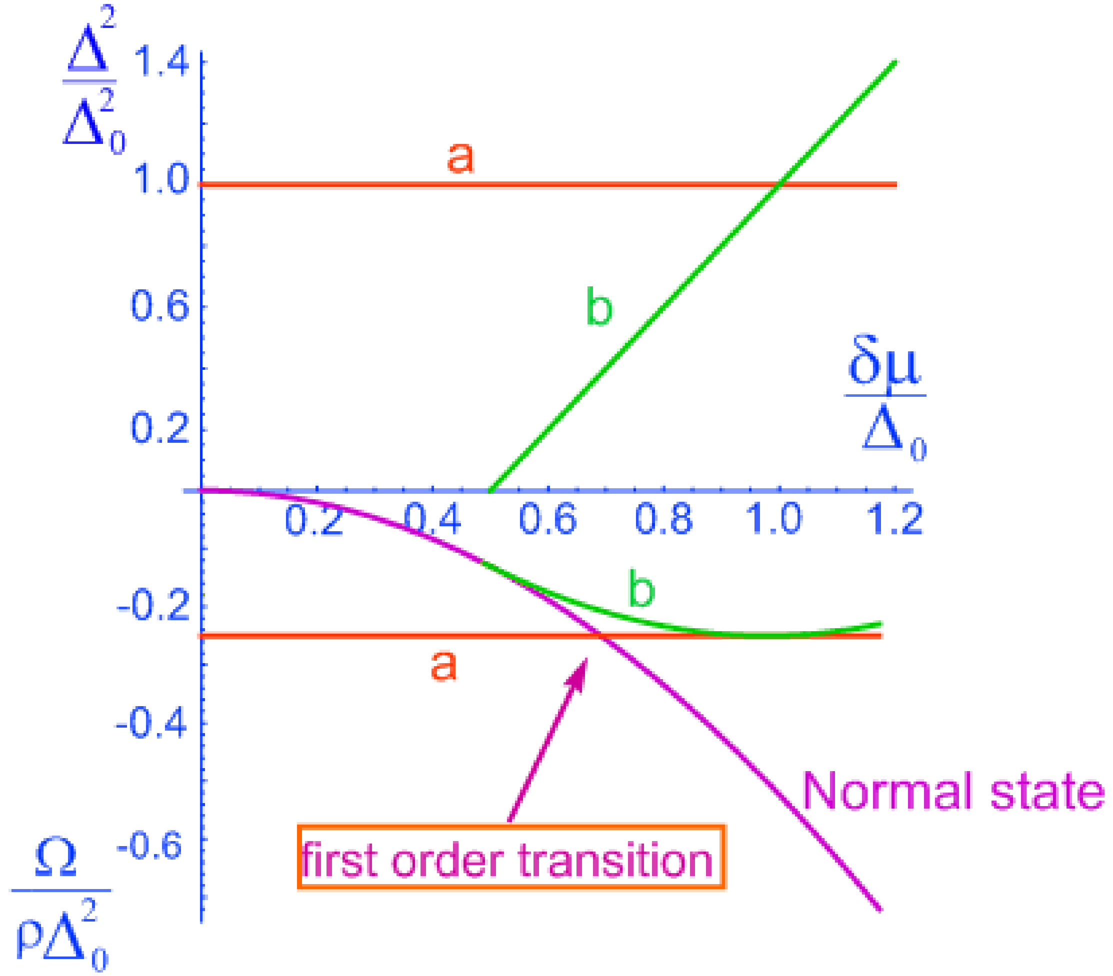
and the gap equation (for spin up and down fermions):

$$\Delta = ig\int \frac{d\vec{v}}{4\pi}\frac{\mu^2}{\pi}\int \frac{d^2\ell}{(2\pi)^2}\frac{\Delta}{(\ell_0 - \delta\mu)^2 - \ell_{\parallel}^2 - \Delta^2}$$

This has two solutions:

$$a):\ \Delta = \Delta_0, \quad b):\ \Delta^2 = 2\delta\mu\,\Delta_0 - \Delta_0^2$$

Favored solution

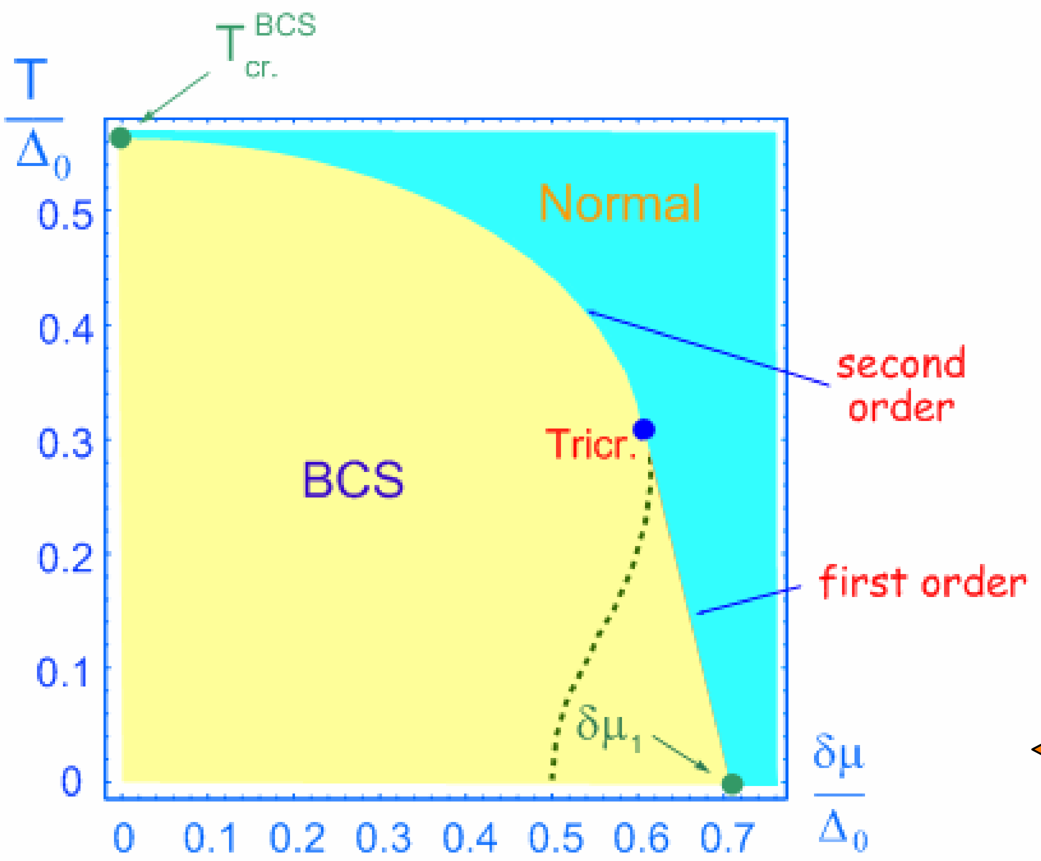


$$\boxed{\Delta = \Delta_0}$$

Also: $$\Omega_\Delta - \Omega_0(\delta\mu) = -\frac{\rho}{4}\left(-2\delta\mu^2 + \Delta_0^2\right)$$

First order transition to the normal state at

$$\boxed{\delta\mu = \delta\mu_1 = \frac{\Delta_0}{\sqrt{2}}}$$



For constant Δ, Ginzburg-Landau expanding up to Δ^6